Topical: Tractor atom interferometer for fundamental physics and space science applications

Primary Author: Georg Raithel, University of Michigan, Ann Arbor, MI

Phone: 734 647 9031, email: graithel@umich.edu

Co-Authors: Vladan Vuletic, Massachusetts Institute of Technology, Cambridge, MA

Vladimir S. Malinovsky, US Army Research Laboratory, Adelphi, MD

Tractor atom interferometer for fundamental physics and space science applications

Georg Raithel, University of Michigan, Ann Arbor, MI

Vladan Vuletic, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA

Vladimir S. Malinovsky, US Army Research Laboratory, Adelphi, MD

1 Overview

1.a Objective. In a collaborative effort between three research groups, we propose to design and build a novel type of tractor atom interferometer for inertial sensing of rotation (gyroscope), acceleration (accelerometer), gravity (gravity gradiometer), and high precision timing (clock). The collaboration is led by Prof. G. Raithel (experiment, U of M), Prof. V. Vuletic (experiment, MIT), and Dr. V. S. Malinovsky (theory, ARL). The ultimate goal of the research proposal is to develop quantum detectors that go beyond the "Standard Quantum Limit (SQL)" for uncorrelated atomic states, approaching the more fundamental "Heisenberg limit" – a potential increase of the signal-to-noise ratio by multiple orders of magnitude. The proposed quantum detectors are based on uninterrupted, three-dimensional confinement and shuttling of atoms in atom-interferometric tractor potentials, and on the use of quantum correlated "spin-squeezed" atomic states (or, more generally, states with increased Fisher information). The proposed development will provide a substantial sensitivity enhancement of atom-interferometer-based inertial sensors and next-generation atomic clocks, enabling capabilities in space exploration as well as inertial navigation in GPS-denied environments.

1.b Team. Prof. G. Raithel (experiment, U of M) is an expert in various types of atomic traps, atom guides, and interacting many-body systems. Prof. V. Vuletic (experiment, MIT) is an expert in laser cooling and trapping, quantum optics, quantum entanglement, spin squeezing, high-precision spectroscopy, and quantum information processing. Dr. V. S. Malinovsky (theory, ARL) is an expert in quantum control theory and its applications to quantum sensing, atom interferometry, quantum information, and metrology. Synergy between these areas of expertise is crucial for the successful realization of the proposed research and development of quantum technology.

1.c Beneficiary areas. The precision of atom interferometers (AIs) enables the search for dark matter, the detection of gravitational waves, and inertial navigation. There is considerable untapped potential in further advances within these devices. Improvements in atomic clocks would significantly enhance the long-term stability of navigation and timekeeping in situations where communication and resynchronization are limited. AI-based gravity gradiometers can reach sensitivities of several orders of magnitude beyond conventional gravity sensors, allowing them to detect dense materials in shipping containers or underground structures. We believe that AI-based sensors for navigation, fundamental physics research, and remote probing of celestial bodies via AI-based "geo"desy will play a crucial role in future NASA missions. High-precision AIs also have dual applications that are crucial for the success of future DoD missions.

2. AI background

Since their first demonstrations [1-4], AIs [5,6] have become a powerful tool with a broad range of applications in fundamental physics, e.g., testing the equivalence principle, free fall and (non)-Newtonian forces [7-14], gravitational-wave detection [15], precision measurements of atomic constants [16-18], and applied science, e.g., inertial sensing [19-21] and geodesy [22,23]. Previous work on AI includes free-space [24-26] and point-source [27, 28] AI, as well as guided-wave AI experiments [29-31] and proposals [32, 33]. Free-space and point-source AIs typically employ atomic fountains or dropped / freely expanding atom clouds. The point-source method supports efficient readout and data reduction [34], enables high bandwidth, and affords efficiency in the partial-fringe regime. Atomic fountains typically employed in free-space AI maximize interferometric time and thus increase sensitivity [24-26], but may require large experimental setups. Guided-wave AIs offer compactness and are often used as Sagnac rotation sensors, but are susceptible to noise in the guiding potentials. In both free-space and guided-wave AI, wave-packet dynamics along unconfined degrees of freedom can cause wave-packet dispersion and failure of the split wave-packets to recombine. Coherent recombination of split atomic wave-functions upon their preparation and time-evolution remains challenging in recent AI studies [35-38]. Atom interferometry is a cornerstone of space-based fundamental and applied research in the cold-atom lab (CAL [39]), where decoherence due to guide- and trap-induced forces and apparatus-size issues, otherwise encountered due to free fall, are significantly reduced. Wave-packet dispersion and atomic interactions, as well as practical problems associated with efficient closure control still remain even at CAL and its successors. Here, we propose tractor atom interferometry (TAI) and atom entanglement to address critical limitations.

3. TAI concept

TAI differs from cold-atom free-space, point-source, and guided-wave AI in that the interfering atomic wave-packet components are transported in conservative, sub-micron to mm-sized, three-dimensional (3D) traps that are formed by tractor potentials that move on pre-determined trajectories. *At all times* during the AI sequence, the AI wave-function components are given by the 3D center-of-mass ground states of the tractor traps. The latter can be implemented via optical tweezers (tractor beams), optical lattices, RF-dressed potentials, optical or magnetic potentials on atom chips, etc., and any combination of these. Uninterrupted 3D confinement in tractor traps (1) guarantees recombination, (2) allows arbitrary holding times, directional reversal, complex trajectory patterns for cancellation of unwanted sensitivities, and (3) addresses signal degradation caused by wave-function dispersion and limitations in recombination control.

In TAI, the tractor controls (laser-beam angles, diameters, powers and phases, electric and magnetic fields) define the pre-determined trajectories $\vec{x}_i(t)$ of the tractor-potential minima, marking the centers of the tractor traps. A pair of traps for *i=1, 2* are intersecting at initial and final space-time points $\vec{x}_{init}(t_{init})$ and $\vec{x}_{final}(t_{final})$. This situation, while classically forbidden, can be realized by employing a pair of different internal quantum states with different, state-specific tractor traps that coincide at the initial and final space-time points. AI beam-splitters are implemented via coupling-laser pulses or other quantum beam splitters. The interferometric phase of the TAI is $\Delta = \frac{1}{\hbar}(S_1 - S_2)$, where the trajectories $S_i = \int_{x_{init},t_{init}}^{x_{final},t_{final}} L(\vec{x}_i(t), \frac{d}{dt}\vec{x}_i(t), t)dt$ are given by the pre-determined trajectories of the tractor traps. The dependencies of the differential interferometric phases on rotation and acceleration scale as

$$\Delta\varphi = \frac{m\,a\,KzT}{\hbar} \quad \text{and} \quad \Delta\varphi = \frac{2mK\vec{\Omega}\cdot\vec{A}}{\hbar} \quad .$$

Here, *m* is the atom mass, *a* is the acceleration, $\vec{A}$ is the interferometric area, $\vec{\Omega}$ is the frame's angular velocity measured against an inertial frame, *K* is the number of loops in the TAI sequence, *z* is the well separation along the acceleration vector, and *T* is the AI time. A discussion of quantum-projection-limited sensitivity levels for rotation and acceleration is provided in [40]. Estimates for the conditions of the present proposal are given in Section 6.

TAI differs from other work on cold-atom free-space, point-source, and guided-wave implementations of AI (see Section 2) in that the interfering wave-function components are confined in 3D *at all times*, suppressing dispersion and allowing for maximum control. Proper programming of the tractor traps ensures AI closure. TAI is quite robust against background inertial effects and uncontrolled wave-packet dispersion. Geometry and speeds of the TAI tractor trajectories are user-programmable and flexible, including multi-loop designs, trap-hold intervals, and twisted patterns. Forces of constraint follow from the programmed trajectories and must remain within the limits given by the tractor trapping forces.

The concept translates well to microgravity implementations, where the tractor-trap depth can be relaxed into the sub-Hz regime, which largely eliminates concerns with trap-depth variations, the AI time *T* can extend to minutes, which leads to greatly enhanced sensitivities, and the motional time scale of the atoms in the tractors becomes so slow that technical noise in the acoustic and higher-frequency bands does not couple into the center-of-mass state of the atoms.

4. Overcoming the Standard Quantum Limit in TAI

For *N* measurements or a measurements with *N* independent (uncorrelated) particles, the precision of any interferometer improves as $1/\sqrt{N}$. The SQL can be overcome by performing a TAI Ramsey sequence using a quantum correlated (entangled) state of the many particles, rather than an uncorrelated state, as the input state for the measurement sequence. Particularly simple and robust entangled states are so-called spin squeezed states [41-43], where the quantum noise is redistributed between two different quadratures. Spin squeezing can be induced by coupling the atoms to the light field inside an optical cavity, thereby creating an effective interaction between distant atoms (cavity spin squeezing) [42]. Spin squeezing has been demonstrated in clocks [42-44], but so far not in any atomic interferometer, because the atoms need to be well localized in the final readout of the interferometer, which is difficult to achieve in free-space interferometers. However, the TAI is ideally suited for creating and utilizing spin squeezing since the atomic wavefunction remains localized in the optical trapping potential at all times. By coupling the atomic ensemble to an optical cavity during the state preparation and readout it should then be possible to overcome the SQL in a TAI system. The cavity in the design shown in Section 7 can serve the double purpose of enhancing and filtering the tractor light beams, and providing the effective cavity mediated spin-spin interaction between atoms that is necessary for generating entanglement. The potential payoff in signal-to-noise ratio is large, with ~20dB already demonstrated in clock systems [43], and 50dB of improvement being available at the Heisenberg limit for 10^5 atoms. We will strive to achieve more than 25 dB of improvement over the SQL, corresponding to a factor 300 reduction in averaging time to reach a given precision.

5. Outline of the proposed work

In high-g environments, realizing a TAI by means of optical lattices allows scaling to a large number of atoms, maximizing the signal-to-noise ratio. Here we first propose a theoretical investigation and ground-based experimental realization for

> (1) Validation of the TAI concept in cavity-based optical lattices of rubidium-87 atoms that allow many-atom parallelization in order to raise the quantum-projection-based signal-to-noise level (SQL).
>
> (2) Theoretical design and experimental testing of methods to advance from the SQL towards Heisenberg-limited signal-to-noise via the entanglement of multiple atoms in pairs of interfering TAI tractor wells, using an additional cavity mode.

(3) Application of spin squeezing to the TAI concept and demonstration of the first atom interferometer that operates beyond the SQL.

The ground-based research platform will allow for the development and testing of prototype tools and methods, validation of critical TAI and entanglement concepts, and development of protocols for rapid and near-adiabatic TAI beam splitters and re-combiners. The research will ideally lead to prototypes for applications on high-g dual-use platforms, where large dynamic ranges and sensor detection bandwidths are critically important. An optical-lattice setup with near-780-nm laser beams is expected to work well in high-g conditions with optical-lattice tractor traps that are several 100 kHz deep.

On the ground-based platform, trap-depth noise is expected to limit sensitivities, even in “magic’’ TAI routines with symmetric tractors for the interfering wave-function components. There will generally be a push towards longer-spatial-period traps with reduced well-to-well tunneling rates and with tractor depths below 1 kHz. Remaining limitations due to tunneling, acceleration-induced sag, and non-adiabatic effects can be addressed effectively by moving into microgravity, where there is virtually no limit as to how shallow and de-compressed a TAI tractor trap can be. The trap-depth reduction naturally reduces AI noise caused by trap-depth fluctuations, and the atomic center-of-mass motion de-couples from acoustic and higher-frequency noise.

Therefore, the proposed work proceeds to a microgravity implementation of TAI with super-relaxed, mm-sized tractor potential wells. These will be several orders of magnitude larger in trap size and well-to-well separation than in the ground-based research platform. Millimeter-sized traps remove performance limitations caused by atom tunneling, even at the ultimately targeted trap depths in the sub-Hz regime. The proposed space-based research platform differs from the ground-based lattice approach in that it utilizes only one tractor-trap pair or a small number of such pairs, instead of tens of thousands. Therefore, the proposed space-based platform employs a small number of discrete optical tractor beams with mm-sized beam waists and electro-optic tractor-beam controls. A Rb-87 Bose-Einstein condensate (BEC) is initially adiabatically loaded and decompressed into a single TAI well. The fundamental wave-function modes in the super-relaxed TAI well can range from the 100-micron into the 10-mm range, enabling virtually atom-interaction-free BECs with tens of thousands of atoms. The TAI interference in the space-based platform employs splitting and recombination of a single TAI well into two wells and back into one, and interference of the coherently split and recombined BEC. Trap-depth ramping will enable super-relaxed traps at times when the wave-function components are at their largest separation, where they may be put into a temporary holding pattern to allow for extreme de-compression. The project components targeting space-based TAI are

(4) Validation of a binary-well TAI concept in the ground-based platform with a pair of discrete tractor wells loaded with a Rb-87 split-BEC wave-function. The tractors are formed with a small set of optical-tweezer beams and optical-guiding tubes.

(5) Furnishing of plans for a similar apparatus with larger, further-relaxed tractor wells loaded with a split-BEC wave-function.

(6) Theoretical and experimental investigation of entanglement methods to (partially) advance from the SQL towards Heisenberg-limited signal-to-noise in binary-well TAI implementations.

(7) Realization of an experimental TAI prototype with super-relaxed wells and sub-SQL operation by squeezing. Ideally, this could be on a space-based vehicle to allow for space

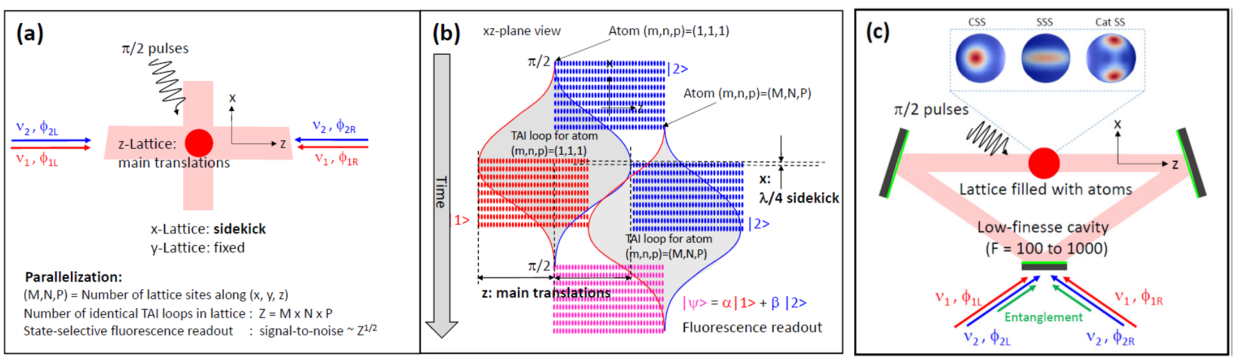
applications. For prototyping purposes and technology research, it will also be possible to utilize modern user platforms such as the Einstein Elevator at the DLR site in Hannover.

6. Expected immediate and broader impact

The extreme shallowness and large size of the super-relaxed TAI wells and the virtual absence of tunneling promise high levels of performance to address topics in the areas of fundamental physics and in low-g navigation, addressed in Section 2. "Magic" optical-tractor wells with depths in the sub-Hz regime, prepared with intensity-levelled trapping beams, are expected to afford splitting times exceeding several minutes, as well as macroscopic distances between coherently split wave-function components. These capabilities, combined with dispersion-free, 3D uninterrupted wave-function confinement shared among all TAI implementations, and with atom entanglement to move towards the Heisenberg quantum-sensing limit, form prerequisites for future transformative progress in AI.

For a performance estimate of acceleration sensitivity, we use $T=100$ s and $z=0.1$m and a phase resolution of 2π/100 to find $\delta a = 5\times10^{-13}\,g$, which improves to $\delta a = 3\times10^{-14}\,g$ after 25 dB of squeezing. These figures surpass state-of-the art sensitivities ($\sim 10^{-9}\,g/\sqrt{Hz}$) due to the long hold time afforded by TAI and the squeezing. For sensitivity in angular frequency, we assume an area of $A=0.01\text{m}^2$, a phase resolution of 2π/100 and $K=10$ loops (which can be traversed within 100 s) to find a resolution $\delta\Omega = 2\times10^{-10}\,\text{rad/s}$, which improves to $\delta\Omega = 1\times10^{-11}\,\text{rad/s}$ after 25 dB of squeezing. Those estimates also compare fairly well with the state of the art.

7. Selected technical components



Figures (a) and (b) shows concepts of a basic ground-based implementation. The lattice spin states are the $F=1$ (red) and $F=2$ (blue) m=0 clock levels of Rb-87, which are trapped in respective 3D optical lattices. The y-lattices are both static and overlapped at all times. The x-lattices feature a short-distance differential "sidekick" between $F=1$ and $F=2$ that displaces the lattice-trapped $F=1$ and $F=2$ wave-function components from each other by half a lattice period. This detail suppresses collisions during the TAI sequence. The z-translations form a large AI area in the space-time plane in (b) used for inertial sensing (gray areas; shown for two of the $\eta M \times N \times P$ atoms; filing factor η). The AI sequence involves π/2 AC-shift-free microwave or Raman pulses to open and close the interferometer. The AI signal is acquired via imaging readout after completion of the TAI loops.

Figure (c) shows a cavity-based implementation. The depicted 3-mirror cavity is operated bi-directionally with color-dependent phase-, amplitude and frequency control to effectuate tractor transport along the cavity axis and mode-filtering to realize pure, 3D, Hermite-Gaussian optical tractor potentials. Cavity-assisted entanglement using a third color, depicted in green, may employ the same or a different cavity. Several examples of squeezed spin states are shown on top.

References

[1] O. Carnal and J. Mlynek, “Young’s double-slit experiment with atoms: A simple atom interferometer,” Phys. Rev. Lett. 66, 2689–2692 (1991). http://dx.doi.org/10.1103/PhysRevLett.66.2689

[2] D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, “An interferometer for atoms,” Phys. Rev. Lett. 66, 2693–2696 (1991). http://dx.doi.org/10.1103/PhysRevLett.66.2693

[3] F. Riehle, Th. Kisters, A. Witte, J. Helmcke, and Ch. J. Bordé, “Optical Ramsey spectroscopy in a rotating frame: Sagnac effect in a matter-wave interferometer,” Phys. Rev. Lett. 67, 177–180 (1991). http://dx.doi.org/10.1103/PhysRevLett.67.177

[4] M. Kasevich and S. Chu, “Atomic interferometry using stimulated Raman transitions,” Phys. Rev. Lett. 67, 181–184 (1991). http://dx.doi.org/%2010.1103/PhysRevLett.67.181

[5] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, “Optics and interferometry with atoms and molecules,” Rev. Mod. Phys. 81, 1051–1129 (2009). http://dx.doi.org/%2010.1103/RevModPhys.81.1051

[6] S. Abend, M. Gersemann, D. Schubert, C. Schlippert, E. M. Rasel, M. Zimmermann, M. A. Efremov, A. Roura, F. A. Narducci, and W. P. Schleich, “Atom interferometry and its applications,” in Proceedings of the International School of Physics “Enrico Fermi”, edited by W. P. Schleich, E. M. Rasel, and S. Wolk (IOS Press, Amsterdam, 2020). https://ebooks.iospress.nl/pdf/doi/10.3254/978-1-61499-937-9-345

[7] M. G. Tarallo, T. Mazzoni, N. Poli, D. V. Sutyrin, X. Zhang, and G. M. Tino, “Test of Einstein equivalence principle for 0-spin and half-integer-spin atoms: search for spin-gravity coupling effects,” Phys. Rev. Lett. 113, 023005 (2014). http://dx.doi.org/10.1103/PhysRevLett.113.023005

[8] D. Schlippert, J. Hartwig, H. Albers, L. L. Richardson, C. Schubert, A. Roura, W. P. Schleich, W. Ertmer, and E. M. Rasel, “Quantum test of the universality of free fall,” Phys. Rev. Lett. 112, 203002 (2014). http://dx.doi.org/%2010.1103/PhysRevLett.112.203002

[9] T. Kovachy, P. Asenbaum, C. Overstreet, C. A. Donnelly, S. M. Dickerson, A. Sugarbaker, J. M. Hogan, and M. A. Kasevich, “Quantum superposition at the halfmetre scale,” Nature 528, 530 (2015). http://dx.doi.org/10.1038/nature16155

[10] M. Jaffe, P. Haslinger, V. Xu, P. Hamilton, A. Upadhye, B. Elder, J. Khoury, and H. M¨uller, “Testing subgravitational forces on atoms from a miniature in-vacuum source mass,” Nat. Phys. 13, 938 (2017).

[11] G. Rosi, G. D’Amico, L. Cacciapuoti, F. Sorrentino, M. Prevedelli, M. Zych, C. Brukner, and G. M. Tino, “Quantum test of the equivalence principle for atoms in coherent superposition of internal energy states,” Nat. Comm. 8, 15529 (2017). http://dx.doi.org/10.1038/nphys4189

[12] J. B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich, “Atom interferometer measurement of the Newtonian constant of gravity,” Science 315, 74–77 (2007). http://dx.doi.org/%2010.1126/science.1135459

[13] V. Ménoret, P. Vermeulen, N. Le Moigne, S. Bonvalot, P. Bouyer, A. Landragin, and B. Desruelle, “Gravity measurements below 10−9 g with a transportable absolute quantum gravimeter,” Sci. Rep. 8, 12300 (2018). http://dx.doi.org/%2010.1038/s41598-018-30608-1

[14] V. Xu, M. Jaffe, C. D. Panda, S. L. Kristensen, L. W. Clark, and H. Müller, “Probing gravity by holding atoms for 20 seconds,” Science 366, 745–749 (2019). http://dx.doi.org/10.1126/science.aay6428

[15] Jason M. Hogan and Mark A. Kasevich, “Atominterferometric gravitational-wave detection using heterodyne laser links," Phys. Rev. A 94, 033632 (2016). http://dx.doi.org/10.1103/PhysRevA.94.033632

[16] D. Hanneke, S. Fogwell, and G. Gabrielse, “New measurement of the electron magnetic moment and the fine structure constant,” Phys. Rev. Lett. 100, 120801 (2008). http://dx.doi.org/%2010.1103/PhysRevLett.100.120801

[17] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Müller, “Measurement of the fine-structure constant as a test of the Standard Model,” Science 360, 191–195 (2018). http://dx.doi.org/10.1126/science.aap7706

[18] L. Morel, Z. Yao, P. Cladé, and S. Guellati-Kh´elifa, “Determination of the fine-structure constant with an accuracy of 81 parts per trillion,” Nature 588, 61–65 (2020). http://dx.doi.org/10.1038/s41586-020-2964-7

[19] B. Barrett, P. Cheiney, B. Battelier, F. Napolitano, and P. Bouyer, “Multidimensional atom optics and interferometry,” Phys. Rev. Lett. 122, 043604 (2019). http://dx.doi.org/10.1103/PhysRevLett.122.043604

[20] K. Bongs, M. Holynski, J. Vovrosh, P. Bouyer, G. Condon, E. Rasel, C. Schubert, W. P. Schleich, and A. Roura, “Taking atom interferometric quantum sensors from the laboratory to real-world applications,” Nat. Rev. Phys. 1, 731 (2019). http://dx.doi.org/10.1038/s42254-019-0117-4

[21] C. L. Garrido Alzar, “Compact chip-scale guided cold atom gyrometers for inertial navigation: Enabling technologies and design study,” AVS Quantum Science 1, 014702 (2019). http://dx.doi.org/10.1116/1.5120348

[22] Y. Bidel, N. Zahzam, A. Bresson, C. Blanchard, M. Cadoret, A.V. Olesen, and R. Forsberg, “Absolute airborne gravimetry with a cold atom sensor,” J. Geod. 94, 20 (9 pp.) (2020). http://dx.doi.org/10.1007/s00190-020-01350-2

[23] G.M. Tino, A. Bassi, G. Bianco, Kai Bongs, P. Bouyer, L. Cacciapuoti, S. Capozziello, Xuzong Chen, M.L. Chiofalo, A. Derevianko, W. Ertmer, N. Gaaloul, P. Gill, P.W. Graham, J.M. Hogan, L. Iess, M.A. Kasevich, H. Katori, C. Klempt, Xuanhui Lu, Long-Sheng Ma, H.Muller, N.R. Newbury, C.W. Oates, A. Peters, N. Poli, E.M. Rasel, G. Rosi, A. Roura, C. Salomon, S. Schiller, W. Schleich, D. Schlippert, F. Schreck, C. Schubert, F. Sorrentino, U. Sterr, J.W. Thomsen, G. Vallone, F. Vetrano, P. Villoresi, W. von Klitzing, D. Wilkowski, P. Wolf, Jun Ye, Nan Yu, and Mingsheng Zhan, “Sage: A proposal for a space atomic gravity explorer,” Eur. Phys. J. D 73, 228 (20 pp.) (2019). http://dx.doi.org/10.1140/epjd/e2019-100324-6

[24] P. Asenbaum, C. Overstreet, T. Kovachy, D. D. Brown, J. M. Hogan, and M. A. Kasevich, “Phase shift in an atom interferometer due to spacetime curvature across its wave function,” Phys. Rev. Lett. 118, 183602 (2017). http://dx.doi.org/%2010.1103/PhysRevLett.118.183602

[25] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino, "Precision measurement of the Newtonian gravitational constant using cold atoms," Nature 510, 518 (2014). https://doi.org/10.1038/nature13433

[26] P. Hamilton, M. Jaffe, J. M. Brown, L. Maisenbacher, B. Estey, and H. M¨uller, "Atom interferometry in an optical cavity," Phys. Rev. Lett. 114, 100405 (2015). https://link.aps.org/doi/10.1103/PhysRevLett.114.100405

[27] S. M. Dickerson, J. M. Hogan, A. Sugarbaker, D. M. S. Johnson, and M. A. Kasevich, "Multiaxis inertial sensing with long-time point source atom interferometry," Phys. Rev. Lett. 111, 083001 (2013). http://dx.doi.org/%2010.1103/PhysRevLett.111.083001

[28] G. W. Hoth, B. Pelle, S. Riedl, J. Kitching, and E. A. Donley, "Point source atom interferometry with a cloud of finite size," Appl. Phys. Lett. 109, 071113 (2016). http://dx.doi.org/10.1063/1.4961527

[29] S. Wu, E. Su, and M. Prentiss, "Demonstration of an area-enclosing guided-atom interferometer for rotation sensing," Phys. Rev. Lett. 99, 173201 (2007). http://dx.doi.org/%2010.1103/PhysRevLett.99.173201

[30] E. R. Moan, R. A. Horne, T. Arpornthip, Z. Luo, A. J. Fallon, S. J. Berl, and C. A. Sackett, "Quantum rotation sensing with dual Sagnac interferometers in an atomoptical waveguide," Phys. Rev. Lett. 124, 120403 (2020). http://dx.doi.org/%2010.1103/PhysRevLett.124.120403

[31] "Atom- interferometry using Kapitza-Dirac scattering in a magnetic trap," R. E. Sapiro, R. Zhang, and G. Raithel, Phys. Rev. A **79**, 043630 (2009). https://doi.org/10.1103/PhysRevA.79.043630

[32] J.P. Davis and F.A. Narducci, "A proposal for a gradient magnetometer atom interferometer," J. Mod. Opt. 55, 3173 (2008). http://dx.doi.org/10.1080/09500340802468633

[33] M. Zimmermann, M.A. Efremov, W. Zeller, W.P. Schleich, J.P. Davis, and F.A. Narducci, "Representation-free description of atom interferometers in time-dependent linear potentials," New J. Phys. 21, 073031 (2019). http://dx.doi.org/10.1088/1367-2630/ab2e8c

[34] Y.-J. Chen, A. Hansen, M. Shuker, R. Boudot, J. Kitching, and E.A. Donley, "Robust inertial sensing with point-source atom interferometry for interferograms spanning a partial period," Opt. Express 28, 34516–34529 (2020). http://dx.doi.org/10.1364/OE.399988

[35] J. A. Stickney and A. A. Zozulya, "Wave-function recombination instability in cold-atom interferometers," Phys. Rev. A 66, 053601 (2002). http://dx.doi.org/%2010.1103/PhysRevA.66.053601

[36] J. A. Stickney and A. A. Zozulya, "Influence of nonadiabaticity and nonlinearity on the operation of cold-atom beam splitters," Phys. Rev. A 68, 013611 (2003). http://dx.doi.org/10.1103/PhysRevA.68.013611

[37] J. H. T. Burke, B. Deissler, K. J. Hughes, and C. A. Sackett, "Confinement effects in a guided-wave atom interferometer with millimeter-scale arm separation," Phys. Rev. A78, 023619 (2008). http://dx.doi.org/10.1103/PhysRevA.78.023619

[38] J. A. Stickney, R. P. Kafle, D. Z. Anderson, and A. A. Zozulya, "Theoretical analysis of a single- and double-reflection atom interferometer in a weakly confining magnetic trap," Phys. Rev. A 77, 043604 (2008). http://dx.doi.org/10.1103/PhysRevA.77.043604

[39] K. Frye, S. Abend, W. Bartosch, A. Bawamia, D. Becker, H. Blume, C. Braxmaier, Sheng-Wey Chiow, M.A. Efremov, W. Ertmer, P. Fierlinger, T. Franz, N. Gaaloul, J. Grosse, C. Grzeschik, O. Hellmig, V.A. Henderson, W. Herr, U. Israelsson, J. Kohel, M. Krutzik, C. Kurbis, C. Lammerzahl, M. List, D. Ludtke, N. Lundblad, J.P. Marburger, M. Meister, M. Mihm, H. Muller, H. Muntinga, A.M. Nepal, T. Oberschulte, A. Papakonstantinou, J. Perovsek, A. Peters, A. Prat, E.M. Rasel, A. Roura, M. Sbroscia, W.P. Schleich, C. Schubert, S.T. Seidel, J. Sommer, C. Spindeldreier, D. Stamper-Kurn, B.K. Stuhl, M. Warner, T. Wendrich, A. Wenzlawski, A. Wicht, P. Windpassinger, N. Yu, and L.Worner, "The Bose-Einstein condensate and cold atom laboratory," EPJ Quantum Technol. 8, 38 pp. (2021). http://dx.doi.org/10.1140/epjqt/s40507-020-00090-8

[40] A. Duspayev and G. Raithel, "Tractor atom interferometry," Phys. Rev. A 104 (2021). http://dx.doi.org/10.1103/PhysRevA.104.013307

[41] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, "Spin squeezing and reduced quantum noise in spectroscopy," Phys. Rev. A 46, R6797 (1992). https://link.aps.org/doi/10.1103/PhysRevA.46.R6797

[42] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletic, "Implementation of Cavity Squeezing of a Collective Atomic Spin," Phys. Rev. Lett. 104, 073602 (2010). https://link.aps.org/doi/10.1103/PhysRevLett.104.073602

[43] O. Hosten, N. Engelsen, R. Krishnakumar, and M. Kasevich, "Measurement noise 100 times lower than the quantum-projection limit using entangled atoms," Nature 529, 505-508 (2016). https://doi.org/10.1038/nature16176

[44] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletic, "Orientation-Dependent Entanglement Lifetime in a Squeezed Atomic Clock," Phys. Rev. Lett. 104, 250801 (2010). https://link.aps.org/doi/10.1103/PhysRevLett.104.250801