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Cover Letter

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I am honoured to write you in vew to publish my white paper entitled :

A NEW LIMIT TO THE CORE MASS IN STARS WITH $M \geq 2M_{\Theta}$

In wainting for a favourable response want, to accept my best greetings.

Signed : The author :

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A NEW LIMIT TO THE CORE MASS IN STARS WITH $M \geq 2M_{\Theta}$

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Abstract: According to the studies of (Schönberg & Chandrasekhar 1942; Henrich & Chandrasekhar 1941)[1,2], it exists an upper limit to the mass of the isothermal core for the stars situated on the post main sequence MS on the HR diagram with a mass $M \geq 2M_{\Theta}$. In the present work, and using another approach that I find more rigorous than the calculus done in the other works, I demonstrate the existence of an other value to this upper limit and I establish in function of this upper limit M_{iso} the formulae of the luminosity produced by these stars.

THE WHITE PAPER/

For stars of masses greater than $2M_{\Theta}$ and classified within the post main sequence on the HR diagram, the interior region or the core is under the rule of the gravitational contractions in the phase of hydrogen rarefaction. Because of the lack of the hydrogen, the luminosity produced by the core is null (L =0) and therfore the core is isothermal. In this phase, the gravitational energy generated from these contractions in the core heats the upper layers, and this increasing in temperature in these layers allows the nuclear reactions to take place in the so called circum-nuclear shell situated above the core. This shell feeds the core with the nuclear reaction products and contribuate in increasing the mass of this core. Henrich & Chandrasekhar (1941) [1]and Schönberg & Chandrasekhar (1942) [2] calculated the greatest mass M_{iso}supported by this isothermal core. In the present work, I find an other upper limit to this mass and in function of this mass I establish the formulae of the luminosity produced by these stars in the frame of the following approach:

In this work I find that the value of M_{iso} for which the pressure in the core is maximum $dP_{iso}/dM_{iso}=0$ is given by:

$$M_{iso}=\left[\frac{3}{4\pi}\left(\frac{15}{11}\right)^{3}\right]^{\frac{1}{2}}\left(\frac{1}{Gm_H}\right)^{3/2}\left(\frac{kT_{iso}}{\mu_{iso}\rho_{iso}^{1/3}}\right)^{3/2}\cong 0.778\left(\frac{1}{Gm_H}\right)^{3/2}\left(\frac{kT_{iso}}{\mu_{iso}\rho_{iso}^{1/3}}\right)^{3/2}$$

The maximum value of pressure corresponding to this mass is then given by:

$$P_{iso|\max}=\frac{8}{11}\left(\frac{kT_{iso}\rho_{iso}}{\mu_{iso}m_H}\right)= =\frac{8}{15}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}G\ \rho^{\frac{4}{3}}\ M_{iso}^{2/3}$$

We define the P_{env}^{iso} as the pressure at the interface between the isothermal core and the envelope and M is the whole mass of the star.

Approximately, one finds(see http://personalpages.to.infn.it/~ferrari/fda/cap13.pdf , page 349) :

$$P_{env}^{iso} \cong \frac{G}{8\pi\langle r^4\rangle}\left(M^2 - M_{iso}^2\right)$$

Confronting now the two pressures P_{env}^{iso} and $P_{iso|\max}$ to find the relation between the mass

of the whole star M and the mass of the isothermal core of the same star M_{iso} which corresponds to the maximum of pressure $P_{iso|\max}$ in the core, I find:

$$P_{iso|\max} = P_{env}^{iso} \quad \Rightarrow \frac{8}{11}\frac{kT_{iso}\rho_{iso}}{\mu_{iso}m_H} = \frac{81}{4\pi}\frac{1}{G^3M^2}\left(\frac{kT_{iso}}{\mu_{env}m_H}\right)^4$$

Therefore the allowed values of M_{iso} are given by:

$$M_{iso} \le 0.261\left(\frac{\mu_{env}}{\mu_{iso}}\right)^2 M \qquad \text{which verify the condition } P_{iso|\max} \ge P_{env}^{iso}.$$

This is the result obtained in this present work. The calculus of this upper mass by L. R. Henrich and S. Chandrasekhar [1,3,4] and Schönberg & Chandrasekhar [2] gives the following result:

$M_{iso} \le 0.35(\mu_{env}/\mu_{iso})^2 M$ which is given by $M_{iso} \le 0.35M$ for $\mu_{env}/\mu_{iso} = 1$.

It appears that the two approaches to estimate the value of M_{iso} don't diverge and

although they give different values the results aren't very far from each other.

THE CALCULUS OF THE LUMINOSITY PRODUCED BY THE ENVELOPE:

Since the luminosity produced in the isothermal core is null $\left(L(r,\ 0 \le r \le R_{iso}) = 0\right)$, I can assume the luminosity produced by such stars to be equal to:

$$L = \int_{R\ iso}^{R} 4\pi\ r^2 \rho_{env}^{iso}\ \varepsilon(r) dr$$

where $\varepsilon(r)$ is the energy produced by a mass unit in the envelope, ρ_{env}^{iso} is the density of the envelope assumed to be quasi constant as mentioned above: $\rho_{env}^{iso} \approx cte$

R and R_{iso} are respectively the radius of the star and the radius of the isothermal core.

If $\varepsilon(r)$ can be supposed constant and approximated such that [5], page 89:

$$\varepsilon(r) \approx \varepsilon \cong \frac{1}{4\pi R^2 \rho_{env}^{iso}} \left(\frac{dL}{dr}\right)_{r=R}$$

Taking this expression of $\varepsilon(r)$, the luminosity of such stars is then given by:

$$L \cong \frac{1}{R^2}\left(\frac{R^3}{3} - \frac{R_{iso}^3}{3}\right)\left(\frac{dL}{dr}\right)_{r=R}$$

where R_{iso} is related to the mass of the isothermal core M_{iso} by $R_{iso} = \left(3M_{iso}/4\pi\rho\right)^{1/3}$ which can be approximated to be equal to:

$R_{iso} = \left(3M_{iso}/4\pi\bar{\rho}\right)^{1/3}$ where $\bar{\rho}$ is the mean density of the star, it's given by $\bar{\rho} = M \Big/ \frac{4}{3}\pi R^3$

Then: $L \cong \left(1 - \frac{M_{iso}}{M}\right)\frac{R}{3}\left(\frac{dL}{dr}\right)_{r=R}$ If the energy transport in the envelope is radiative, the luminosity $L(r)$ is so that [5], page 89

$$L(r) = -\frac{4acT^3}{3\kappa\rho} 4\pi\ r^2 \left(\frac{dT}{dr}\right)_{rad}$$

where a is the radiative constant, c is the light velocity, κ is the opacity, and $\left(dT/dr\right)_{rad}$ is the radiative temperature gradient. From this we obtain:

$$\frac{dL(r)}{dr} = -\frac{4ac}{3\kappa\rho} 8\pi\ rT^3\left(\frac{dT}{dr}\right)_{rad} \ - \ -\frac{4ac}{3\kappa\rho} 4\pi\ r^2\left[3T^2\left(\frac{dT}{dr}\right)_{rad} + T^3\left(\frac{d^2T}{dr^2}\right)_{rad}\right]$$

$$\frac{dL(r)}{dr} = \frac{4ac}{3\kappa\rho}\ \frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3}\left(\frac{\mu m_H}{k}\right)\rho^{1/3} \times \left[\left(\frac{64\pi^2 r^3 T^3 \rho + 48\pi^2 r^4 T^2 \rho}{M^{1/3}}\right) \ - \left(\frac{64\pi^3 r^6 T^3 \rho^2}{3M^{4/3}}\right)\right]$$

Then the expression of the luminosity becomes:

$$L \cong \left(1 - \frac{M_{iso}}{M}\right)\frac{R}{3}\ \frac{4ac}{3\kappa\rho}\ \times \frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3}\left(\frac{\mu m_H}{k}\right)\rho^{1/3}\ \times \left[\left(\frac{32\pi^2 r^3 T^3 \rho + 48\pi^2 r^4 T^2 \rho + 32\pi^2 r^3 T^3 \rho}{M^{1/3}}\right) - \left(\frac{64\pi^3 r^6 T^3 \rho^2}{3M^{4/3}}\right)\right]$$

This relation provide us with an estimation of the luminosity produced by a star from the

POST MS region with a mass $M \geq 2M_{\Theta}$ in which the envelope is considered radiative. To calculate the luminosity of the star given by this relation , I use the value of the molecular weight μ given by [6], page 119 and the opacity κ given by [6], page 119.

The comparison between the observational luminosities and those calculated in the present work.

Stars	The Luminosity calculated $L_{CALCULATED}\ (erg\ s^{-1})$	Bolometric Luminosity $L_{MEASURED}\ (erg\ s^{-1})$	$\frac{L_{CALCULATED}}{L_{MEASURED}}$
RR LYR.	4.793×10^{35}	5.030×10^{35}	0.953
CAPELLAB	4.045×10^{36}	4.800×10^{35}	8.427
β Cephei	1.053×10^{35}	2.215×10^{36}	0.047
SU.CAS	7.194×10^{36}	1.012×10^{36}	7.109
SZ.TAU	3.076×10^{37}	1.561×10^{36}	19.109
SU.CYG	2.557×10^{37}	1.809×10^{36}	14.135
RT.AUR	3.788×10^{37}	1.776×10^{36}	20.329
T.VUL	7.782×10^{37}	2.215×10^{36}	35.133
POLARIS	1.153×10^{38}	2.096×10^{36}	55.00

References:

[1]-Henrich, L. R., & Chandrasekhar S. 1941, M. N., 94, 525.

[2]-Schönberg, M., & Chandrasekhar S. 1942, ApJ, 96, Vol. 2, 161.

[3]-Henrich, L. R., & Chandrasekhar S. 1929, M. N., 89, 739.

[4]-Henrich, L. R., & Chandrasekhar S. 1936, M. N., 96, 179.

[5]-Unno, W., Osaki, Y., Ando, H., Saio, H., Shibahashi, H. 1989, Nonradial oscillations of stars, (2nd ed.; university of Tokyo press)

[6]-Eddington, A.S., & Chandrasekhar, S. 1988, The internal constitution of the stars, (Cambridge University Pres)