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Topical: Gravitational wave detection with optical atomic clocks in space

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I. Introduction and motivation - With the numerous gravitational wave (GW) events detected by the LIGO/VIRGO collaboration over the past 5 years[1–3], gravitational wave astronomy has emerged as a powerful tool with which to study the universe, illuminating previously invisible astrophysical phenomena. While there is no doubt that LIGO and other terrestrial detectors will observe many additional GW events in the coming years, terrestrial detectors are only sensitive to GWs with frequencies above ∼10 Hz, due to seismic and Newtonian noise [1, 4]. The desire to observe a wider range of astrophysical phenomena has motivated proposals for larger scale space-based GW detectors [4–11].

Existing and proposed space-based GW detectors can be broadly classified into two main categories. The first are optical interferometric detectors analogous to LIGO in space, such as the proposed Laser Interferometer Space Antenna (LISA) [4], which will be composed of three spacecraft forming either a two or three arm Michelson interferometer, with roughly equal length arms to reduce susceptibility to laser frequency noise. These GW detectors detect signals in a broad frequency band with sensitivities determined by the detector arm length, the photon flux, and acceleration noise of the satellites [4]. In order to change the frequency range over which an interferometric detector is maximally sensitive, the detector transfer function must be modified by physically re-positioning the satellites [12].

A second class of space-based GW detectors rely on stable internal frequency references, such as Doppler tracking of distant spacecraft [13–16]. These detectors search for changes in the frequency of electromagnetic waves due to effective Doppler shifts arising from passing GWs. Doppler tracking of spacecraft has been successfully employed to set the existing limits on milliHertz gravitational wave events [13–16]. Because the sensitivities of this class of detector are generally limited by the stability of the frequency reference rather than the photon flux [16, 17], there is a clear motivation to improve the internal frequency references used for GW detection through the adoption of atomic physics techniques, such as either atomic interferometry (AI) [5–8], or optical lattice atomic clocks [10, 18]. In both cases the atoms serve as stable phase or frequency references [19]. The distinction between the AI and optical lattice clock techniques therefore primarily lies in whether or not the atoms are strongly confined. In the case of AI detector schemes the atoms are unconfined and in an inertial reference frame, eliminating the need for a separate drag-free test mass, but requiring picoKelvin atomic temperatures and larger satellites to enable larger atomic wavefunction spatial separations [5–8]. In contrast, in the optical lattice clock approach the atoms are strongly confined in a lattice, relaxing the satellite size and atom temperature requirements but requiring a drag-free test mass [18]. For the remainder of this white paper we choose to focus on the optical lattice clock approach. We do so motivated in large part by the fact that while there are already efforts to realize ground- and space-based AI GW detectors such as MAGIS [20], ELGAR [21], AEDGE [22], and AION [23], there are currently no comparable scale efforts to explore clock-based GW detectors in space. Nevertheless, much of the discussion here is equally applicable to AI-based detectors.

The performance of optical atomic clocks has accelerated rapidly in recent years [24–28], achieving remarkable levels of precision and becoming increasingly sensitive to relativistic phenomena [29–31]. This has led to the recent achievements of differential clock comparisons at stabilities below the level of 1×10^{-17} at 1 s [32, 33], and precision below 1×10^{-20}, enabling the observation of the gravitational redshift due to Earth's gravity across a single mm scale atom ensemble [33]. These advances motivate the application of optical lattice atomic clocks to GW detection [10, 18, 19]. One obvious motivation for the integration of optical clocks with space-based optical interferometric GW detectors is that they can provide the best possible laser frequency stability for optical interferometry. However, optical atomic clocks can also be used as independent, complementary detectors, potentially integrated on the same satellites as optical interferometer GW detectors or in their own dedicated constellations.

Atomic-clock-based GW detection requires ultra-cold atoms loaded in optical lattices on at least two satellites. These atomic references are each compared to a local laser, and the lasers are phase-locked to each other over an optical link between the satellites. The strain from a GW will induce relative motion between the two satellites that is imprinted as an effective Doppler shift on the photons sent across the optical link that can be measured be performing a synchronized differential clock comparison. Importantly, such a design allows for tuning of the detection window by varying the local control sequences applied to the atoms, without requiring reconfiguration of the optical baseline length/satellite positions [18, 19]. Different sequences have been proposed that can tune the detector transfer function to achieve optimal sensitivity to GWs with baselines that are either longer or shorter than the GW wavelength [18, 19, 34]. Atom-based detectors are therefore of particular advantage for mid-band gravitational wave astronomy [11], providing the ability to lock onto and track GW signals from merger events over the duration of their chirp, which can last for days or even several years in the relevant frequency range, up to GW frequencies of ∼10 Hz, where they would become detectable by terrestrial detectors such as LIGO.

This frequency range is also of relevance to searches for light scalar dark matter, to which atom-based detectors are uniquely sensitive [35, 36]. Some models of dark matter result in variations of the fine-structure constant, which will thereby cause the atomic clock transition frequencies to vary in both space and time and would result in a gravitational wave-like oscillatory signal. Thus atomic clocks can serve as tunable narrow-band detectors with complementary features to optical interferometry and an additional, alternative method to detect GW and dark matter signals, all while being integrated onto the same satellites and providing improved laser frequency stability. We believe this motivates consideration for investment in improving optical clock portability, robustness, and performance, and consideration of the incorporation of optical clocks in future detector designs.

II. Detector concept - The GW detection scheme we focus on in this white paper was originally proposed in Ref. [18], and we briefly review it here. A constellation of two or more satellites are connected over optical links formed by lasers transmitted and collected with conventional optical telescopes. Each satellite contains an optical lattice atomic clock, a drag-free reference mass, and a clock laser referenced to a stable optical cavity. The clock laser in each satellite can be phase-locked to the light received from any other satellite, such that the two lasers function as a single ultra-stable clock laser shared between them. In synchronized clock comparisons offset by the time required for the laser light to travel from one satellite to the other, any laser frequency noise will be common mode for the two measurements and can subsequently be rejected. This method of laser frequency noise rejection has recently been used to demonstrate quantum-projection-noise-limited synchronized differential clock comparisons between spatially separated ensembles in a geometry closing resembling a miniaturized version of a GW detector configuration, achieving atom-atom Ramsey coherence times of 26 s, a stability of $9.7(4) \times 10^{-18}$ at 1 s, and an ultimate precision of $8.9(3) \times 10^{-20}$ after only 3.3 hours of averaging using a ~ 1 Hz linewidth rack-mounted clock laser [32]. These results indicate that synchronous differential clock comparisons really can completely cancel laser frequency noise, relaxing the requirements on the stable optical cavities required in the satellites. In addition, because the laser frequency noise is cancelled by the differential atom comparison rather than in two arms of the detectors as in LIGO or LISA, clock-based detectors can operate in networks of satellites with highly heterogenous baseline lengths.

The recorded differential signal in a two-satellite configuration due to a GW of frequency f_{GW} and wavelength λ_{GW} is of the form $s(t) \propto \sin\left(\frac{\pi d}{\lambda_{GW}}\right) \cdot \mathcal{T}\left(f_{GW}\right)$, where the first term is the physical detector transfer function due to the distance d between the detectors, and $\mathcal{T}$ is the spectral filter function corresponding to the control sequence of the atoms of duration T. As is apparent from the form of the expression, this detector design decouples these two effects and allows the latter to be adjusted by the control sequence locally applied to the atoms in each satellite. For a Ramsey sequence, for example, $\mathcal{T} = \mathrm{sinc}(\pi f_{GW} T)$, but other read-out and control sequences can be applied to effectively vary the sensitivity window without affecting the noise floor, tuning the spectral sensitivity of the detector [18, 19].

The smallest detectable fractional frequency difference σ_{min} between the two clocks, and hence the smallest measurable GW-induced strain, is given by $\sigma_{min}\left(\tau\right) = \frac{\sqrt{2}}{2\pi\nu C T}\sqrt{\frac{T+T_{\mathrm{d}}}{\tau}}\sqrt{\frac{\xi^2}{N}}$, where ν is the transition frequency, T is the interrogation time, T_{d} is the dead time between experiment cycle, τ is the averaging time, N is the atom number per clock per measurement, C is the contrast of the atomic clock state measurement, and ξ is the Wineland squeezing parameter [37–39]. For unentangled atomic states, $\xi = 1$, while for highly entangled states it can approach $\xi = 1/\sqrt{N}$, corresponding to the Heisenberg limit for differential clock stability. A reasonable target strain sensitivity for GW detection in the mid-band range is $\sigma_{min} \approx 1 \times 10^{-20}$ at 1 s. Recently, a clock comparison between two spatially resolved unentangled regions of the same mm scale ensemble of strontium atoms achieved a record differential stability of 4.4×10^{-18} at 1 s and an ultimate precision of 7.6×10^{-21} [33], approaching the level of performance required for detection of some strong GW signals in a space-based detector. Additional gains in stability can be anticipated through further increases in interrogation time T, atom number N, and the introduction of entangled states to reduce ξ and approach the Heisenberg limit. Based on current trends, differential stabilities below 1×10^{-20} appear realistic for ground-based optical lattice clocks within the next decade.

III. Astrophysics objectives - With numerous GW detections by the LIGO/VIRGO collaboration [1, 2] and the anticipated launch of the LISA mission [4], an exciting new era of GW astronomy has begun. Clock-based GW detectors can achieve scientifically relevant GW strain sensitivities in the frequency band between the LISA and LIGO detectors, which here we refer to here as the "mid-band" (approximately 30 mHz to 3 Hz). By operating in the mid-band, a clock-based GW detector will be complementary to both the LISA and LIGO detectors and would greatly advance NASA interests in GW science. The mid-band GW spectrum offers rich scientific potential [11]. Known sources in the mid-band that are expected to have both the strength and abundance to allow for detection include white dwarf binary mergers, intermediate mass black hole mergers, intermediate mass-ratio inspirals, and stellar mass black hole and neutron star binary inspirals. Stellar mass binary inspirals that are initially observed in this band could later be observed by LIGO once they pass into the higher frequencies. Such joint observation would be a powerful new source of information, giving for example a prediction of the time and location of a merger event in LIGO which would allow optical, x-ray, gamma ray, and other telescopes to observe these mergers as they occur. As sources generally remain within the mid-frequency band for an extended duration of time, they can be localized by a single-baseline detector because the detector will change orientation and position significantly during the time spent observing a single source [40]. The mid-band therefore appears to be well suited for localization and prediction of merger events, both because the GW wavelengths are shorter than in the LISA band, and also because the time between detection and merger is typically shorter (months) compared to LISA (years) resulting in many merger events during the mission lifetime.

The identification of the properties of dark matter is a major scientific goal that has the potential to unveil new paradigms in astrophysics, cosmology and particle physics. A single baseline clock-based GW detector is also sensitive to ultra-light scalar dark matter [35, 36]. These type of dark matter candidates can cause temporal oscillations in fundamental constants with a frequency set by the dark matter mass, and amplitude determined by the local dark matter density, resulting in a modulation of atomic transition energies. This signal is optimally measured by comparing two spatially separated atomic clocks referenced by a common laser. Space-based GW observatories would offer significantly longer baselines than similar terrestrial detectors, probing a lower frequency range and therefore a lighter dark matter mass scale. The search for ultra-light scalars through atomic clock GW observatories would complement NASA efforts to constrain the properties of dark matter through CMB measurements, X-ray and gamma-ray telescopes.

IV. Technological impacts - In addition to the astrophysics and fundamental physics objectives discussed in Sec. III, the realization of space-based optical clocks will also have a number of technological impacts on NASA missions and goals, as well as industry and science more generally. The satellites in a GW detector could serve as "master clocks" in a gravitationally unperturbed reference frame [24, 41]. Optical or microwave frequency links to the Earth could thus enable the redefinition of the SI second at the 10^{-18} level using an optical frequency standard. In addition, clock comparisons between terrestrial optical clocks and the GW detector will be sensitive to the height of the Earth-based clocks with respect to the geoid at the centimeter scale [42], complimenting NASA gravitational mapping missions such as GRACE [43]. Furthermore, optical links between the satellites and the ground combined with the link between the two satellites can potentially be leveraged to enable real-time intercontinental optical clock comparisons.

The optical stability provided by the combination of the ultra-stable optical cavity and the strontium optical frequency reference could also be harnessed to test laser ranging [43–45] between the satellites at new levels of sensitivity to absolute length. Accurate absolute ranging between spacecrafts will have applications to gravitational mapping [43] and deep space navigation [44].

Terrestrial navigation will also benefit from the realization of such detectors. Space-based atomic clocks already play a critical role in GNSS. Although the short term positioning accuracy of GNSS networks, such as the Global Positioning System (GPS), is not presently limited by the stability of the vapor cell microwave atomic clocks in the satellites, the clocks require regular hourly corrections from the ground control stations in order for the network to remain synchronized and not accumulate significant errors [24]. Improving the stability of the clocks in GNSS satellites or adding additional space-based optical clock satellite nodes would improve GNSS system integrity. The network would require less frequent ground-to-space based communications for clock correction, and could potentially operate autonomously for days or even weeks [24]. The satellites required for GW detection could serve as optical clock nodes, and will also result in the development of space-qualified transportable optical clocks, which could then be adapted for further integration in GNSS networks.

V. Required advancements in technology - Despite rapid progress, a significant gap remains between optical atomic clocks operating in research laboratories and those currently deployed in real-world environments. Metrology laboratories focus on high stability and absolute accuracy, resulting in meter-scale clocks operated by highly trained scientists. These laboratory-based devices typically occupy multiple optical tables, consume kW-levels of electrical power, require cooling water, and employ large vacuum chambers with bulky pumps. Portable versions currently under development retain much of the complexity and size, weight, and power (SWaP) of their laboratory counterparts (e.g., $\sim$500 L volume, $\sim$150 kg mass, $\sim$100 W power consumption for the lasers alone, and independent, disconnected breadboards linked together by a multitude of optical and electrical cables).

For optical clocks to be practical for real-world applications, including eventual deployment in satellites as required for GW detection, the SWaP of the core components (e.g. atomic reference, optical cavity) must decrease significantly. More critically, a system-level approach to the apparatus design is necessary to reduce overall complexity, efficiently route optical beams and electrical signals, and to ensure compatibility among components. Considerable investments and long-term research efforts will be required to realize space-based optical lattice clocks performing at or better than the levels currently demonstrated in research laboratories on Earth.

VI. Ground-based experiments/technology development - As discussed in Sec. V, GW detection with clocks requires a number of advancements in space-ready cold-atom equipment. However, in addition to hardware development, there are a number of other techniques that must be developed for high precision optical lattice clock operation in space. For example, one of the most important aspects of these devices is the confinement provided by the lattice, which decouples the atom's finite temperature and motion from its internal degrees of freedom. This means that atoms cannot be allowed to tunnel from one lattice site to another during clock interrogation, because otherwise the clock transition will be broadened and distorted [46]. The current generation of terrestrial OLCs take advantage of a combination of gravity and moderate lattice depths to prevent tunneling [32, 33, 47]. However, space-based clocks will not be able to take advantage of the gravitational potential to suppress tunneling, and the large optical powers required for deep traps inhibit the design of compact, mobile, low power clocks, and result in Raman scattering out of the excited clock state, limiting coherent interrogation times. New techniques will must be explored and developed to achieve 10 s scale coherence times for optical lattice clocks in space, such as the use of novel lattice geometries [48], the introduction of external gradients, or modulation of the lattice to suppress the effects of tunneling [49].

Target GW strain sensitivities of $< 1\times10^{-20}$ at 1 s will also require further advances in the number of atoms simultaneously interrogated within optical lattice clocks, the realization of clocks capable of zero-dead time operation [50–52], and most likely the use of locally entangled atomic states to enhance the differential stability of the clock comparisons below the quantum projection noise limit. In addition, the design of atom-based GW detection pulse sequences, data analysis techniques, and detector satellite configurations for specific science targets remain nascent and must be explored and developed.

VII. Summary - Optical lattice clocks in space offer the promise of tunable narrowband single or multiple baseline detectors for mid-band gravitational wave detection. The performance of optical clocks continues to rapidly improve. However, additional advances in clock performance are required to achieve the desired levels of sensitivity, and significant advances in optical lattice clock portability and hardiness must to be made to realize space-based operation. A concerted and long-term ground-based campaign to explore this promising but still speculative concept is required in order to realize its potential.

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